NON-GAUSSIAN CHARACTERISTICS OF WAVES IN FINITE WATER DEPTH

BY

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NON-GAUSSIAN CHARACTERISTICS OF WAVES IN FINITE WATER DEPTHS

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This paper presents the results of a study on non-Gaussian characteristics and statistical prediction of peaks, troughs and peak-to-trough excursions of coastal waves. Through the statistical analysis of more than 500 records obtained during the growing stage of a storm, it is found that the parameters associated with skewness and kurtosis involved in the non-Gaussian probability distribution are significant for predicting wave characteristics. These parameters are expressed as a function of water depth and sea severity. The limiting sea severity above which wind-generated coastal waves are considered to be non-Gaussian is obtained. The effect of water depth and sea severity on the statistical characteristics of wave period are clarified. The probability density functions applicable to peaks, troughs, and peak-to-trough excursions of waves in finite water depths are developed based on the non-Gaussian probability distribution. Numerical computations of the newly developed probability distribution are carried out at various water depths for various sea severities. The results of computation show that the probability density functions of peaks and troughs both agree reasonably with the histograms constructed from measured data obtained during the ARSLŒ Project.

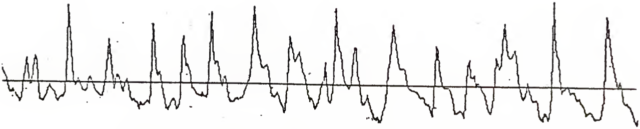
CHAPTER I

INTRODUCTION

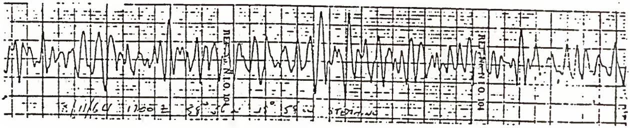
The profile of wind-generated waves observed in the area of finite water depth is significantly different from that observed in deep water, and the difference is pronounced in severe seas. Time histories of the former show a definite excess of high crests and shallow troughs in contrast to waves in deep water as demonstrated in Figure 1.1.

Analyses of wave elevations measured in finite water depth have shown that the histogram of wave elevations is not symmetric with respect to the mean value, and its positive skewness increases with an increase in sea severity for a given water depth. This implies that, from the stochastic point of view, waves in finite water depth can not be considered a Gaussian random process which has been proved acceptable for probabilistic prediction of waves in deep water.

During the past two decades, significant progress has been made in the probabilistic prediction of various characteristics of random waves in deep water. These include prediction of wave height, period, extreme values, breaking phenomenon and grouping characteristics. All of these



Example of wave record observed in finite water depth



Example of wave record observed in ocean

Figure 1.1 Comparison of coastal and ocean wave

prediction methods have been developed based on the Gaussian random process concept. Needless to say, this concept can not be applied for predicting wave characteristics in finite water depth.

Although many studies have been made on wave characteristics in finite water depth, only a limited number of studies has been carried out on a method for predicting statistical properties of wind-generated waves based on the non-Gaussian random process concept. In particular, the extent to which coastal waves deviate from a Gaussian process has not yet been clarified.

The purpose of this study is (1) to clarify the non-Gaussian properties of coastal waves which depend on water depth as well as sea severity, and (2) to develop a method for predicting wave height based on the non-Gaussian random process concept.

To achieve the first goal of this study, a statistical analysis is carried out on wave time histories obtained by the Coastal Engineering Research Center (CERC) during a storm in the Atlantic Ocean Remote Sensing Land-Ocean Experiment (ARSLOE) Project. The wave time histories were obtained at ten locations in water depths ranging from 1.35 m to 24.4 m along the CERC Field Research Facility, Duck, North Carolina.

For the second phase of the study, probability density functions applicable for predicting peaks and troughs are developed separately, since the magnitude of wave peaks in finite water depth is generally much greater than that of troughs. The probability density function of peak-to-trough excursions is then derived assuming that peaks and troughs are statistically independent.

This paper consists of three chapters followed by conclusions and an appendix. Chapter II reviews currently available literature on probability distribution of non-Guassian random processes as well as their application for predicting statistical characteristics of waves in finite water depth.

Chapter III discusses the results of analyzing measured wave data based on the non-Gaussian probability density function. It is found that two parameters associated with skewness and kurtosis involved in the non-Gaussian probability density function are significant for predicting wave characteristics in finite water depth. These two parameters are expressed as a function of sea severity and water depth. The limiting sea severity below which the wind-generated coastal waves are considered to be Gaussian is obtained for a given water depth. The effect of water depth and sea severity on the statistical characteristics of wave period is clarified.

Chapter IV presents the development of a method for predicting peaks and troughs as well as peak-to-trough excursions of waves in finite water depth based on the non-

Gaussian probabiltiy density function. Numerical computations are carried out at water depths ranging from 1.35 m to 8.77 m for various sea severities. The results are compared with measured data obtained during the ARSLOE Project.

Chapter V summarizes the findings and conclusions obtained from the results of the present study.

In Appendix, the wave data used in the present study are described.

CHAPTER II

REVIEW OF LITERATURE

This chapter reviews currently available literatures on the probability distribution applicable to non-Gaussian random waves as a prelude to the present study.

Ochi (1985) has recently presented a state-of-the-art review on stochastic analysis and probabilistic prediction of non-Gaussian random processes in ocean engineering in which he grouped the derivation of the probability density functions applicable to non-Gaussian random process into two categories: one derived from probability theory, the other derived from applying Stokes wave theory. In particular, the latter approach is concerned with wave surface profiles which are expressed as an accumulation of profiles expressed in the form of a series rather than accumulation of simple sinusoidal form on the wave profile. Hence, this approach actually imposes a preliminary form on wave surface profiles.

There are three probability distributions developed by applying the concept of probability theory. These include the Gram-Charlier, Edgeworth, and Longuet-Higgins probability density functions. Gram-Charlier probability density function (Cramer,1946; Kendal and Stuart,1961) is derived from the concept of orthogonal polynomials. Since

Hermite polynomials are orthogonal with respect to the normal distribution, the probability density function is expanded in a series of Hermite polynomials. The Edgeworth series probability density function (Edgeworth 1905) is derived following the same concept as the Gram-Charlier series but the series expansion method is different and the expanded terms are expressed in term of cumulants. On the other hand, the Longuet-Higgins series probability density function (Longuet-Higgins 1963) is derived by applying the cumulants generating function. In his approach, the integrations involved in the derivation are expressed in terms of Hermite polynomials.

It is noted that these three probability distribution functions can be reduced to the same form shown below, although their method of derivation are entirely different.

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{x^2}{2\sigma^2}} \left[1 + \frac{\lambda_3}{3!} H_3\left(\frac{x}{\sigma}\right) + \frac{\lambda_4}{4!} H_4\left(\frac{x}{\sigma}\right) + \frac{\lambda_5}{5!} H_5\left(\frac{x}{\sigma}\right) + \left(\frac{\lambda_6}{6!} + \frac{\lambda_3^2}{72}\right) H_6\left(\frac{x}{\sigma}\right) + \cdots\cdots \right] \tag{2.1}$$

where

σ^2 = variance of x

$\lambda_3 = K_3/(\sqrt{K_2})^3$ = skewness

$\lambda_4 = K_4/(\sqrt{K_2})^4$ = kurtosis - 3

$\lambda_5 = K_5/(\sqrt{K_2})^5$

K_j = cumulants

$H_n(\frac{x}{\sigma})$ = Hermite polynomial of degree n

The first term in the series is the normal distribution; all other terms in the series are attributed to the effect of non-Gaussian characteristics.

Another approach for deriving the probability distributions applicable to non-Gaussian random process is based on the concept that the wave surface profile can be represented approximately by the Stokes wave given by

$$\eta = \frac{1}{2} a^2k + a \cos\chi + \frac{1}{2} a^2k \cos 2\chi + \frac{3}{8} a^3k^2 \cos 3\chi + \ldots \ldots \ldots \qquad (2.2)$$

where

η = wave surface elevation, $a = a(x, t)$ = amplitude

$\chi = kx - wt + \varepsilon$ $\quad\varepsilon$ = random phase

$k = w^2/g$ = wave number

Tayfun (1980) developed the probability density function of wave surface elevations, η, based on Stokes wave 2nd order expansion in deep water without the constant term, $\frac{1}{2} a^2k$, which is the second and third term in

Equation (2.2). By assuming that the amplitude, a, follows Rayleigh distribution and using transformation of random variables, the cumulative distribution function of surface wave elevation, $F(\eta)$, is derived. A numerical computation is required to evaluate the probability density function, $f(\eta)$.

Huang et al.(1983) applied Stokes wave of 3rd order in deep water which is the first four terms in Equation (2.2) to represent the wave surface profile. The probability density function of wave surface elevations, η, is derived by assuming the steepness of wave is very small and then using a perturbation scheme. Huang et al. (1983) further derived the probability density function of η in finite water depth. For this, the Stokes waves of 3rd order in finite water depth is considered. It is given by Bowden (1948) as follows:

$$
\begin{aligned}
\eta = {} & \frac{a^2 k}{2} \coth Kd + a \cos \chi \\
& + \frac{a^2 k}{2} \coth kd \left(1 + \frac{3}{2 \sinh^2 kd} \right) \cos 2\chi \\
& + \frac{3}{8} a^3 k^2 \left(1 + \frac{3}{\sinh^2 kd} + \frac{3}{\sinh^4 kd} + \frac{9}{8 \sinh^6 kd} \right) \cos 3\chi
\end{aligned}
\qquad (2.3)
$$

where

d = water depth

The probability density functions of amplitudes (peaks and troughs) and wave heights have been derived from the same two approaches as considered for the derivation of the probability density function of wave surface elevations: one from probaility theory, the other from applying Stokes wave theory.

Bitner (1980) derived the probability density function of amplitude and wave height by assuming that the wave surface elevation, η, can be expressed as

$$\eta = a \cos (wt - \varepsilon(t)) \tag{2.4}$$

where

$a = a(t)$ = random amplitude, $\varepsilon(t)$ = random phase

Then, by applying Equation (2.1) for the probability density function of η, the probability density function of amplitude is derived. Unfortunately, the probability density function of amplitudes developed by Bitner is not applicable for waves in finite wate depth, since the distributions of peaks and troughs are substantially different, in general, as demostrated in an example shown in Figure 1.1. It is highly desirable to derive the probability density functions applicable for peaks and troughs separately for non-Gaussian random waves.

Arhan and Plaisted (1981) developed the probability density functions of peaks and troughs based on the application of the Stokes wave of 2nd order in finite water depth. They expressed the peaks and troughs of Stokes wave as follows:

$$\xi = a + \varepsilon B\, a^2 \qquad (2.5)$$

where

$\varepsilon = +1$ for peak, -1 for trough,

$$B = \frac{k}{2} \coth kd \left(1 + \frac{3}{2 \sinh^2 kd} \right)$$

d = water depth .

Here, the amplitude, a, follows the Rayleigh distribution.

Tayfun (1984) developed the probability density functions of peaks and troughs by applying the Stokes wave of 2nd order in deep water with a parameter, α, controlling the effect of the nonlinear component. That is,

$$\eta = a \cos X + \frac{1}{2} \alpha k a^2 \cos 2X \qquad (2.6)$$

where

η = wave surface elevation

$0 \leqslant \alpha \leqslant 1$

The peaks and troughs associated with the wave profile shown in Equation (2.6) can be expressed as

$$\xi = \varepsilon a + \frac{1}{2} \alpha k a^2 \qquad (2.7)$$

where

$$\varepsilon = \begin{cases} +1 & \text{for peaks} \\ -1 & \text{for troughs} \end{cases}$$

The amplitude, a, follows the probability distribution developed by Tayfun (1981) which considered the breaking-wave effect.

Tayfun (1983) also developed the probability density function of peak-to-trough excursions of non-Gaussian waves based on the application of Stokes wave of 2nd order in deep water which is the second and third terms in Equation (2.2).

As this brief review of the literature indicates, several studies have been carried out for the developement of probability density functions of wave surface elevations, peaks, troughs, and peak-to-trough excursions based on the Stokes wave theory in representing wave surface profile. This is certainly a way to derive the probability distribution applicable to non-Gaussian random waves. However, this approach imposes a preliminary form as a function of frequency on wave profile which results in a limitation on the non-Gaussian characteristics of random waves in finite water depths. It seems to be desirable to develop the probability density functions of amplitude (peaks or troughs) and wave heights by applying the concept of probability theory, which does not impose any preliminary form on the wave surface profile.

CHAPTER III

NON-GAUSSIAN CHARACTERISTICS OF COASTAL WAVES

As stated in the introduction, the profile of wind-generated waves observed in coastal water shows, in general, a definite excess of high crests and shallow troughs which is significantly different from waves observed in deep water. This difference is particularly pronounced in severe seas. This implies that waves in coastal water where the effect of water depth on wave characteristics is present cannot be considered a Gaussian random process. Hence, clarification of the non-Gaussian characteristics of coastal waves is desirable to provide a basis for statistical prediction of wave characteristics.

The purpose here is to clarify the non-Gaussian properties of coastal waves which depend on water depth as well as sea severity. This chapter is in four sections. The first section summarizes results of the statistical analysis based on the non-Gaussian probability density function. From analysis of more than 500 wave records measured at various water depths during the growing stage of the storm, it is found that two parameters associated with skewness and kurtosis, respectively, involved in the

Longuet-Higgins' non-Gaussian probability density function are significant for predicting wave characteristics in finite water depth. The second section presents the analysis results in which these two parameters are expressed as a function of water depth and sea severity. The third section discusses the limiting sea severity below which wind-generated coastal waves can be considered as a Gaussian random process. The fourth section discusses the statistical characteristics of the period of coastal waves.

3.1 Probability Distribution of Wave Profiles

Several probability density functions have been developed for representing non-Gaussian random processes as outlined in the previous chapter. Since an important objective of the present study is to clarify the extent to which coastal waves deviate from a Gaussian random process, it may be well not to impose any preliminary form on wave profile representation such as Stokes expansion in analyzing data obtained in coastal water. Therefore, the following non-Gaussian probability density function derived by Longuet-Higgins is considered in the statistical analysis of wave profiles.

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{x^2}{2\sigma^2}} \left[1 + \frac{\lambda_3}{3!} H_3\left(\frac{x}{\sigma}\right) + \frac{\lambda_3^2}{72} H_6\left(\frac{x}{\sigma}\right) + \frac{\lambda_4}{4!} H_4\left(\frac{x}{\sigma}\right) + \frac{\lambda_5}{5!} H_5\left(\frac{x}{\sigma}\right) + \cdots\cdots \right] \tag{3.1}$$

where σ^2 = variance of wave deviation from the mean

$\lambda_3 = K_3 / (\sqrt{K_2})^3$

$\lambda_4 = K_4 / (\sqrt{K_2})^4$

$\lambda_5 = K_5 / (\sqrt{K_2})^5$

K_j = cumulants

$H_n(\frac{x}{\sigma})$ = Hermite polynomial of degree n

It is noted λ_3 is the skewness of the random variable x, and λ_4 is equal to the kurtosis minus 3. The value of λ_3 and λ_4 may be positive as well as negative in general; however, positive values are observed for most wave profiles in the coastal water.

In applying the above probability density function for the analysis of coastal waves, the following remarks are made:

(1) In general, the accuracy of a function which is expressed in the form of a series of uniform convergence increases with increase in higher order terms. However,

this is not the case for the probability density function given in Equation (3.1). The results of comparisons between histograms and the probability function have shown that higher order terms do not necessarily yield better agreement. Therefore, it is desirable to examine terms of the series which significantly contribute to the distribution.

(2) The probability density function given in Equation (3.1) at times becomes negative for large negative x depending on the value of σ^2 and the number of terms taken into consideration from the series. However, this will not cause any trouble in practice, since the x value where the function becomes negative is usually outside the range where the histogram exists as will be shown later.

In order to clarify possible questions arising from the above remarks, more than 500 wave records measured at various water depths during the growing stage of the storm during the ARSLOE project are analyzed. Details of wave data used in the present study are summarized in the Appendix.

Figure 3.1 shows, as an example, a comparison between the probability density function given in Equation(3.1) with a histogram of the wave profile obtained at an average water depth of 1.35 m. The significant wave height is 2.10 m which is considered severe for this water depth. Four variations of the probability density are compared with the histogram. These four variations represent, respectively, the first term of the series (which is the normal

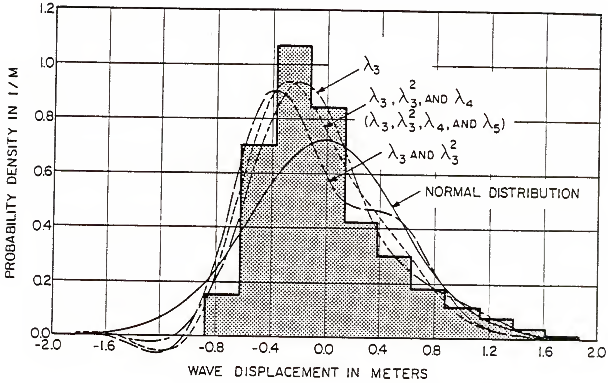


Figure 3.1 Comparison between observed histogram, Gaussian distribution (heavy line), and non-Gaussian distribution:
GAGE 615, average water depth 1.35m,
water depth 2.28 m at 13:15 on Oct. 25th due to tide, significant wave height 2.10 m

distribution), the first two terms (including the parameter λ_3), the first three terms (including the parameters λ_3 and λ_3^2), and the first four terms (including the parameters λ_3, λ_3^2, and λ_4).

As can be seen in this figure,

(1) The histogram deviates substantially from the normal distribution.

(2) The probability density function becomes negative for large negative x. However, the magnitude of the negative probability density is relatively small, on the order of 2 percent or less. Furthermore, the negative density occurs outside the range of the histogram. Therefore, it will not cause any serious problem if we assume this negative probability density to be zero and in turn normalize the entire probability density function so that the area of the density function becomes unity.

(3) The probability density function which includes the parameter λ_3 only agrees reasonably well with the histogram and differs substantially from the normal distribution.

(4) The agreement with the histogram becomes poor if the term with the parameter λ_3^2 is included in addition to the λ_3-term in the probability density function. Although not included in Figure 3.1, the same trend is obtained if the term with the parameter λ_4 is included in addition to the λ_3-term.

(5) The probability density function consisting of the first three terms taking the parameters λ_3, λ_3^2, and λ_4 into consideration agrees well with the histogram.

(6) The addition of the parameter λ_5 to the probability density function does not yield any appreciable change in the shape of the probability density function.

The same trends stated above was also observed for many other wave records obtained at various water depths in various sea severities. Some examples are shown in Figure 3.2(a) through 3.2(g) for which the water depth ranges from 1.35 m to 8.77 m. In each figure, five variations of probability density function are compared with the histogram. These five variations represent different combinations of terms in Equation (3.1). They are

(1) the first term (GAUSS in the figure),

(2) the first two terms (NG-1,triangle marks in the figure),

(3) the first three terms (NG-2, cross marks in the figure),

(4) the first four terms (NG-3, star marks in the figure),

(5) the first five terms (NG-4, square marks in the figure).

From the results shown in the figures, we may safely conclude that the parameter λ_3 which represents the skewness of the wave profile is the dominant parameter affecting the non-Gaussian characteristics of waves in

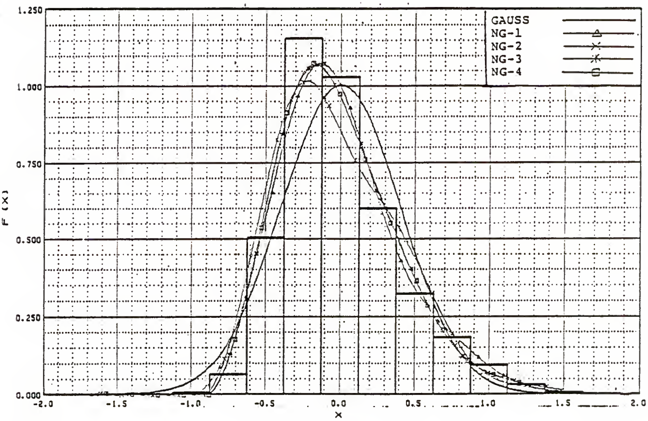


Figure 3.2 Comparison between observed histogram, Gaussian distribution (heavy line), and non-Gaussian distribution:
(a) Average water depth 1.35 m, significant wave height 1.60 m

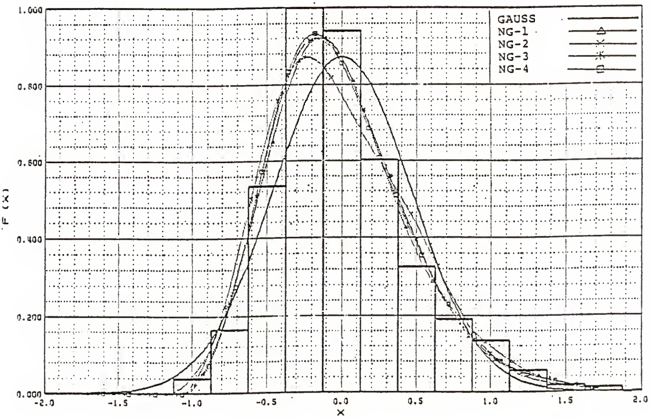


Figure 3.2 (continued)
(b) Average water depth 1.97 m, significant wave height 1.89 m

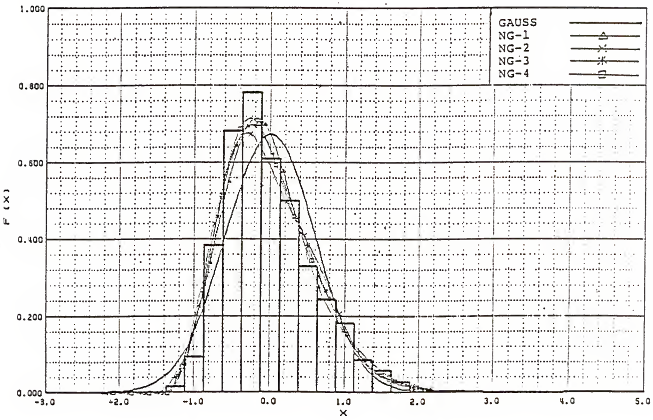


Figure 3.2 (continued)
(c) Average water depth 3.70 m, significant wave height 2.44 m

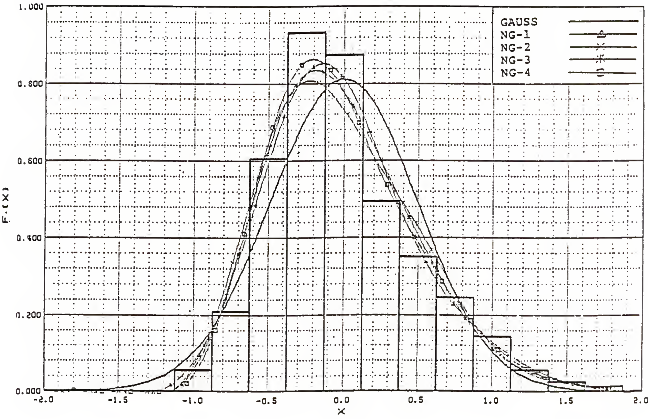


Figure 3.2 (continued)
(d) Average water depth 5.55 m, significant wave height 2.09 m

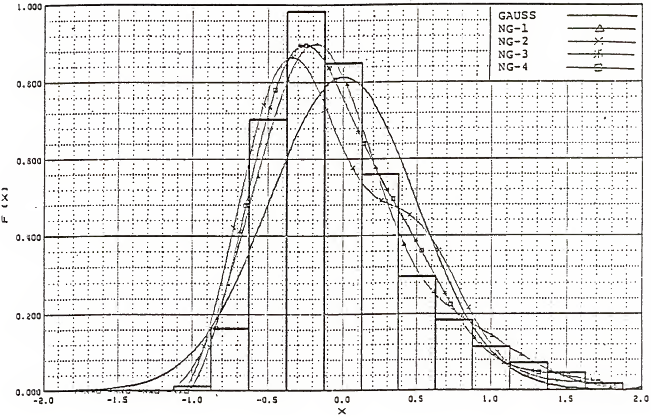


Figure 3.2 (continued)
(e) Average water depth 5.97 m, significant wave height 2.18 m

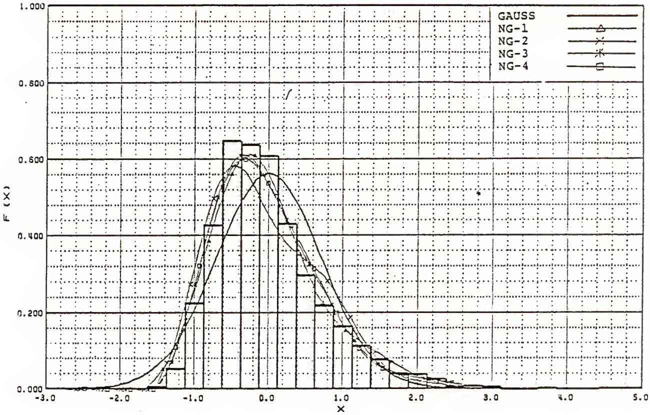


Figure 3.2 (continued)
(f) Average water depth 6.97 m, significant wave height 3.17 m

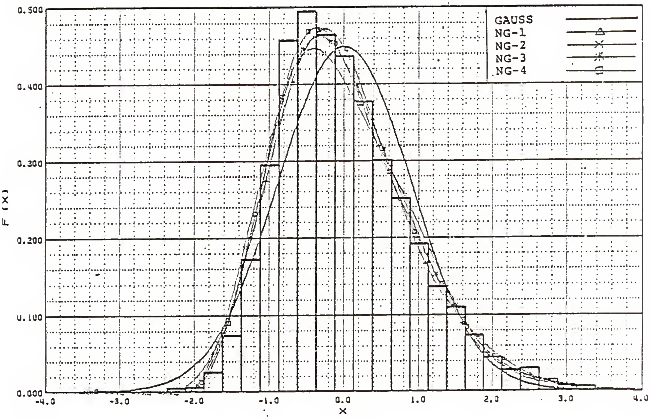


Figure 3.2 (continued)
(g) Average water depth 8.77 m, significant wave height 3.75 m

coastal water and that the combination of the parameter λ_3, λ_3^2, and λ_4 best represents the non-Gaussian probability density function given in Equation(3.1). This conclusion agrees with that made by Huang and Long (1980) in their experimental study of surface elevation probability distribution. On the other hand, the combination of the parameters λ_3 and λ_3^2 or the combination of the parameters λ_3 and λ_4 does not yield satisfactory agreement between the histogram and non-Gaussian probability density function.

Based on the above-mentioned conclusions, only the two parameters λ_3 and λ_4 involved in the non-Gaussian probability density function are considered in the following statistical analysis.

3.2 Analysis of Parameters of the Probability Distribution

It was shown in the previous section that λ_3 and λ_4 in Equation (3.1) are the significant parameters which govern the non-Gaussian probability density function for representing the distribution of wave surface elevations. Since the non-Gaussian characteristics of coastal waves depend on water depth and sea severity, it may be well to examine the properties of these parameters as a function of water depth and sea severity.

It was first thought that the parameter λ_3 (skewness) was a function of significant wave height and wave period.

However, wave records show that λ_3 appears to depend on significant wave height only. To substantiate this, the time variations of significant wave height H_s, parameter λ_3 wave period (average zero-crossing wave period) and water depth obtained at average water depths of 1.35 m and 8.77 m during the growing stage of a storm are shown in Figure 3.3(a) and 3.3(b) respectively.

As can be seen in Figure 3.3(a), the water depth varies with time by a substantial amount due to the tide. This results in an increase and decrease in significant wave height with the same period as the tide (12 hrs.) but the magnitude of significant wave height at high tide increases consistantly during the growing stage of the storm. The time history of the magnitude of the parameter λ_3 (skewness) computed at hourly intervals (with a few points missing due to bad measurement) demonstrates almost the same pattern as that of the significant wave height. However, the magnitude of the zero-crossing average wave period slowly increases with time during the growing stage of the storm.

Another example observed at the average water depth of 8.77 m is shown in Figure 3.3(b). The example shows the water depth still varies with time by a substantial amount due to tide, which is the same pattern observed in the previous example. However, the variation of water depth with time does not result in any significant increase or decrease in the magnitude of significant wave height as it did in the previous example. The magnitude of significant wave height

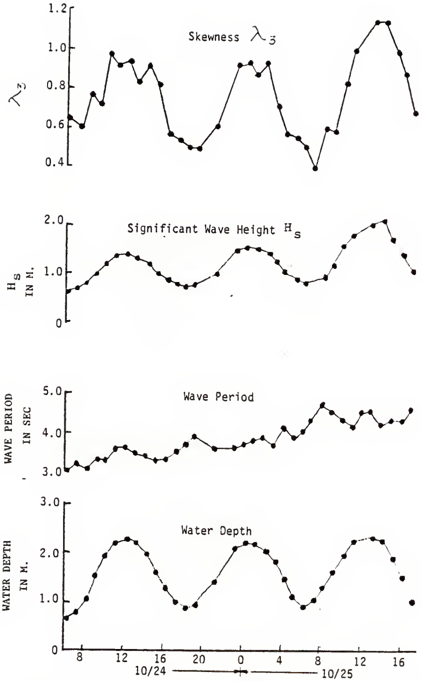


Figure 3.3 Variation of water depth, wave period, significant wave height, and skewness with time:
(a) Average water depth 1.35 m

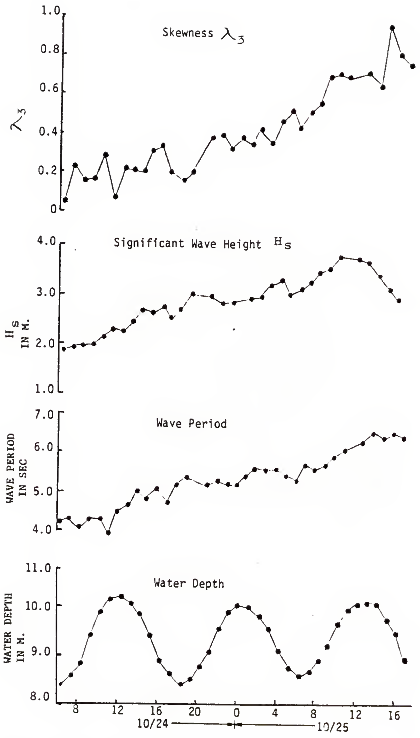


Figure 3.3 (continued)
(b) Average water depth 8.77 m

increases consistently during the growing stage of the storm. A similar trend is also observed in the time variations of parameter λ_3 as well as average zero-crossing wave period.

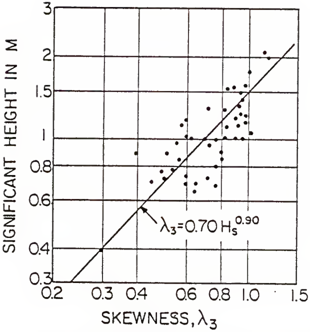
From the results of analysis, several trends are observed. They are

(1) The time variation of water depth due to tide results in a significant variation on the magnitude of significant wave height and parameter λ_3 in shallow water. The significance of this effect is reduced as the average water depth increases.

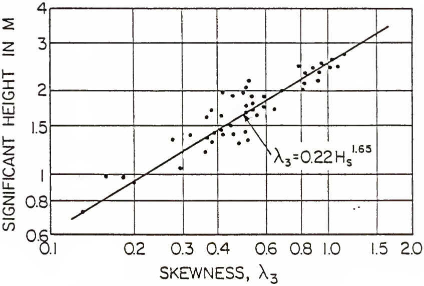
(2) The average zero-crossing wave period slowly increases with time during the growing stage of the storm. This is not affected by the time variation of water depth regardless of the average water depth.

(3) The pattern of time variation of parameter λ_3 (skewness) during the growing stage of a storm is always very similar to that of significant wave height H_s at various average water depths.

Based on these findings, it is apparent that the parameter λ_3 has a functional relationship with significant wave height. The parameter λ_3 computed from wave records measured at various water depths ranging from 1.35 m to 8.77 m is plotted against significant wave height (some examples are shown in Figure 3.4(a) through 3.4(f)). The results presented in these figures are at locations where

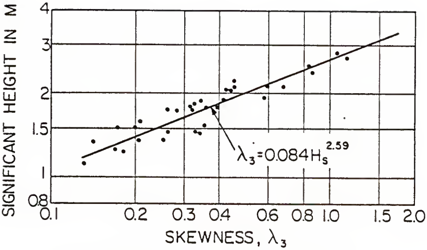


(a) Average water depth 1.35 m

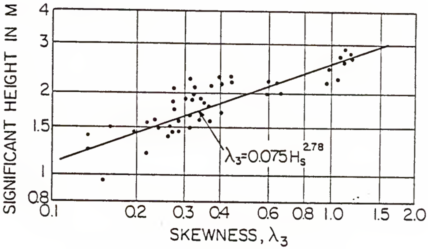


(b) Average water depth 3.70 m

Figure 3.4 Parameter λ_3 as a function of significant wave height

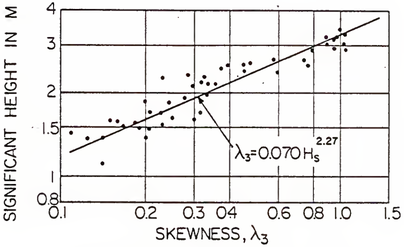


(c) Average water depth 5.55 m

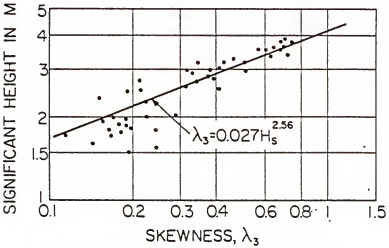


(d) Average water depth 5.97 m

Figure 3.4 (continued)



(e) Average water depth 6.97 m



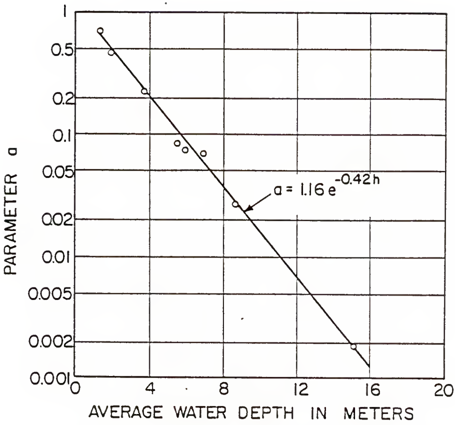
(f) Average water depth 8.77 m

Figure 3.4 (continued)

waves were measured, and the water depth given in each figure is the average water depth during the storm at that location (see Figure A.1 in Appendix). Included in each figure is the functional relationship between λ_3 and significant wave height expressed in the form of $\lambda_3 = a(H_s)^b$, where H is the significant wave height and coefficient "a" and "b" are obtained by drawing the average line through data presented in the figure.

Coefficients "a" and "b" obtained from Figure 3.4(a) through 3.4(f) are then plotted against the average water depth. The results are shown in Figures 3.5(a) and 3.5(b) respectively. These figures show that the values of coefficients "a" and "b" evaluated at a location where the average water depth was in the range of 5.5 m to 6.0 m deviate from the average line drawn in the figure. That is, for a given significant wave height, the skewness evaluated from the record is greater than that evaluated by using the value from the average line. This implies that the water depth of 5.5 m to 6.0 m appears to be the depth where wave breaking occurred during the storm . This has also been mentioned by Aranuvachapun and Thornton (1984) in their study of spatial and temporal transformation of shallow wave energy. Hence, the non-Gaussian characteristics of the waves in this particular location are relatively pronounced.

The coefficients "a" and "b" are expressed as a function of the average water depth h, as shown in Figure



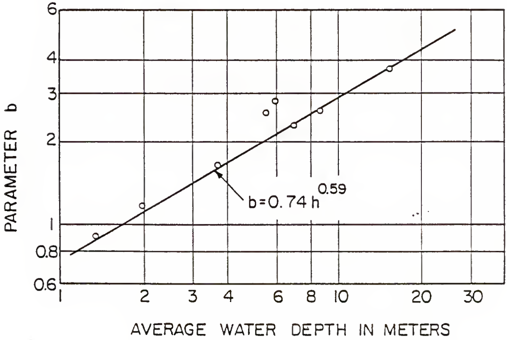


Figure 3.5 Parameter "a" and "b" as a function of water depth

3.5, and thereby the parameter λ_3 can be evaluated in terms of water depth and sea severity as follows:

$$\lambda_3 = 1.16 \exp(-0.42\ h)\ H_s^{0.74\ h^{0.59}} \qquad (3.2)$$

where h = average water depth in meters

H_s = significant wave height in meters.

Next, the parameter λ_4 is evaluated from the records and its values are plotted against the parameter λ_3, as shown in Figure 3.6. The results in the figure show several interesting features:

(1) There is considerable scatter in the value of λ_4 for values of λ_3 less than 0.2. These values are obtained from measured records taken where the water depth is relatively deep, on the order of 15 to 25 meters. Although the values of λ_4 vary considerably for λ_3 less than 0.2, the shape of the non-Gaussian probability distribution appears to be very close to that of the normal distribution. As an example, Figure 3.7 shows a comparison between the non-Gaussian probability density function with $\lambda_3 = 0.2$ and $\lambda_4 = -0.15$ and the normal probability density function in the standardized form . The comparison shows no significant difference between these two probability density functions. From the results of many similar comparisons, it may be

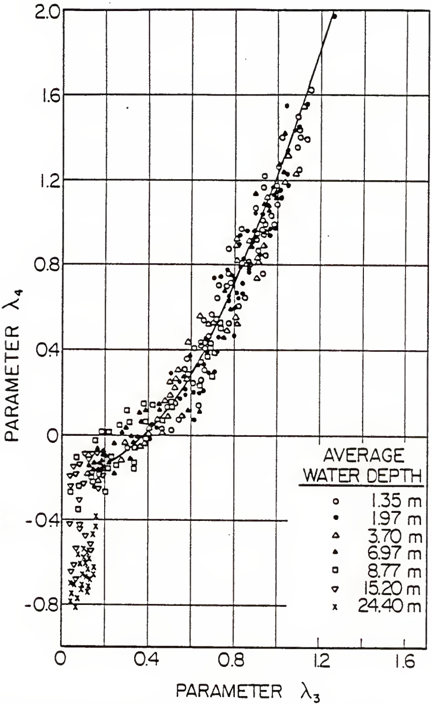


Figure 3.6 Parameter λ_4 as a function of parameter λ_3

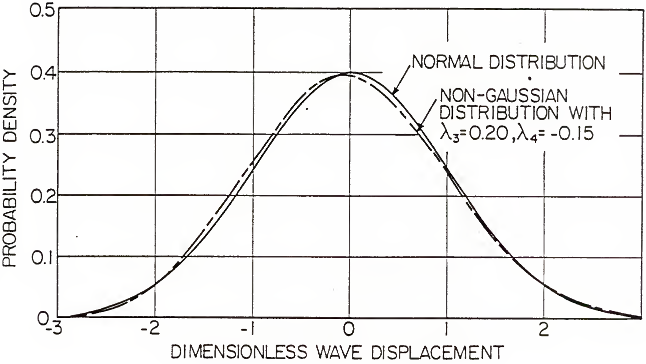


Figure 3.7 Comparison between normal probability distribution and non-Gaussian distribution with $\lambda_3 = 0.20$ and $\lambda_4 = -0.15$

safely concluded that the non-Gaussian presentation of coastal wave profiles (in standarized form) with a value of λ_3 less than 0.2 can be approximated by the Gaussian probability density function (in standardized form).

(2) The largest value of the parameter λ_3 obtained in the present analysis is 1.26 with the parameter λ_4 of 1.98. This is observed in a severe sea of significant wave height 2.31 m at an average water depth of 1.97 m. Figure 3.8 shows the histogram of the wave profile together with a portion of the time series of wave record. As can be seen, the histogram of wave profile deviates from the normal distribution, and the time history of coastal waves show a definite excess of high crests and shallow troughs.

(3)There is some scatter in the λ_4-values for a specified λ_3. For example, the value of λ_4 spreads from 0.45 to 0.9 with an average value of 0.7 for λ_3 =0.8. However, the difference in the λ_4-value for a specified λ_3 does not seriously affect the shape of the non-Gaussian probability density function. Figure 3.9 shows a comparison between three probability density fuctions having the same λ_3-value(0.8), but with values of λ_4 of 0.45,0.70 and 0.90. Included also in the figure is the Gaussian probability density function. The shape of the non-Gaussian probability density functions computed by Equation (3.1) differs significantly from that of the Gaussian probability density function, but there is no substantial difference between the shapes of the non-Gaussian distributions. In other

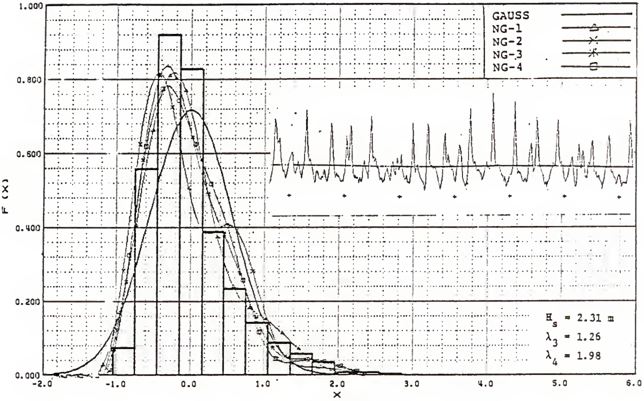


Figure 3.8 Histogram, probability distribution and portion of wave record for significant wave height 2.31 m at average water depth 1.97 m

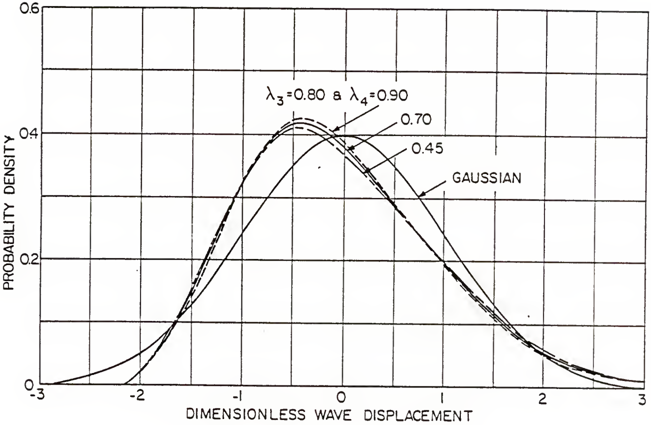


Figure 3.9 Comparison between normal probability distribution and non-Gaussian distribution with λ_3 = 0.80 and λ_4= 0.45, 0.70, and 0.90

words, the scatter in the λ_4 -value for a given λ_3 does not result in any serious difference in the shape of the probability density function.

From the discussion and subsequent conclusion stated in Item (1) and (3) above, the relationship between λ_4 and λ_3 can be formulated by taking the average value of λ_4 for a specified λ_3-value. That is,

$$\lambda_4 = \begin{cases} -0.15 + 1.10\,(\lambda_3 - 0.20)^{1.17} & \text{for } 0.2 \leqslant \lambda_3 \leqslant 0.5 \\ -0.15 + 1.10\,(\lambda_3 - 0.20)^{1.17} + 1.48\,(\lambda_3 - 0.50)^{1.47} & \text{for } \lambda_3 > 0.5 \end{cases} \tag{3.3}$$

In summary, waves at a given water depth may be considered as a Gaussian random process in mild sea if the parameter λ_3 evaluated by applying Equation (3.2) is less than 0.2. However, in severe seas for which the parameter λ_3 is greater than 0.2, waves must be considered as a non-Gaussian random process represented by Equation (3.1) consisting of terms containing the parameters λ_3 , λ_3^2, and λ_4. The parameters λ_3 and λ_4 can be evaluated by Equations (3.2)and (3.3) respectively.

It is of interest to compare the non-Gaussian properties of the measured wave profile obtained in coastal waters with results obtained from nonlinear wave theory. As an example, the skewness λ_3 is computed from deviations from the mean value of nonlinear waves generated by the stream function theory developed by Dean (1974). A total of 40 cases are computed for different wave conditions characterized by the relative water depth, h/L_o, and the relative wave height, H/L_o, where h is the water depth, H is wave height, and L_o is wave length in deep water. The results of the computations are shown in Figure 3.10 where the dashed lines indicating various λ_3-values ranging from 0.3 to 1.5 are generated by connecting values of the same skewness. Included also in the figure are the measured data for λ_3 equal to 0.50, 0.70, and 0.90 obtained from analysis of coastal waves. In plotting these data obtained in random seas in the figure, the significant wave height is considered as H, and the wave length corresponding to the average zero-crossing wave period is considered as L_o.

As can be seen in Figure 3.10, for a given relative wave height, H/L_o, λ_3 based on the stream function theory increases with decrease in relative water depth, h/L_o. This is true for all cases where λ_3 is less than the breaking index curve. However, the measured data do not show the same trend. This may be attributed to the difference in the definition of skewness used in the analysis. Needless to

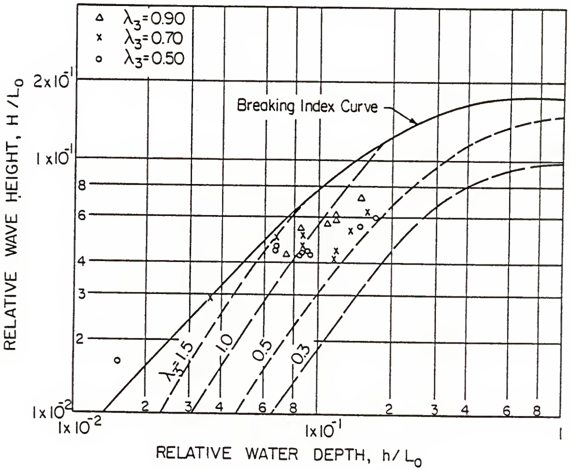


Figure 3.10 Comparison of skewness obtained from measured data with that based on stream function theory

say, in the strict sense skewness should be defined based on a random process. Nevertheless, most of the measured data falls into a zone bounded by relative water depth between 0.06 and 0.20 and relative wave height between 0.04 and 0.07.

3.3 Non-Gaussian Properties and Water Depth

As discussed earlier, the non-Gaussian characteristics of wind-generated coastal waves depend on water depth as well as sea severity. That is, even though the water depth is shallow, waves may still be considered to be a Gaussian random process if the sea severity is mild. Therefore, it may be of considerable interest to find the limiting sea severity below which wind-generated waves are considered to be Gaussian for a given water depth.

In the discussion of Figure 3.7, it was stated that the non-Gaussian presentation of coastal wave profiles with a value of λ_3 less than 0.2 can be approximated by the Gaussian probability distribution, where λ_3 was expressed as a function of water depth and sea severity in Equation (3.2). Therefore, it may be well to confirm whether or not the significant wave height which yields $\lambda_3 = 0.2$ computed from Equation(3.2) for a given water depth represents the limiting sea severity below which the seas can be considered to be a Gaussian random process.

For this, histograms of the wave profiles obtained at various water depths are examined to find the limiting sea severity (significant wave height) below which the histograms approximate a normal distribution. As an example, Figure 3.11 shows the histograms as well as a portion of the wave record obtained in a sea of significant wave height 1.62 m at a location of water depth 6.97 m. This is the largest significant wave height below which Gaussian characteristics were observed during the storm at this location. The computed value of the parameter λ_3 (skewness) is 0.19. A similar analysis was done at various depths and the results are shown in Figure 3.12. Included also in the figure are the lines indicating various λ_3 values computed from Equation (3.2). As can be seen, the largest significant wave height below which Gaussian characteristics are observed for water depths up to 15 m agrees well with the line for $\lambda_3 = 0.2$.

It is noted that the largest significant wave height observed at a water depth of 24.4 m was 4.34 m during the storm, and that all histograms of the wave profile obtained at this location approximate a normal probability distribution. It thus appears that the sea severity at this location did not reach a level during the storm to produce non-Gaussian characteristics. Thus, although confirmation could not be made for a water depth of 24.4 m, it may safely be concluded from Figure 3.12 that coastal waves in seas for which the parameter λ_3 (skewness) is less than

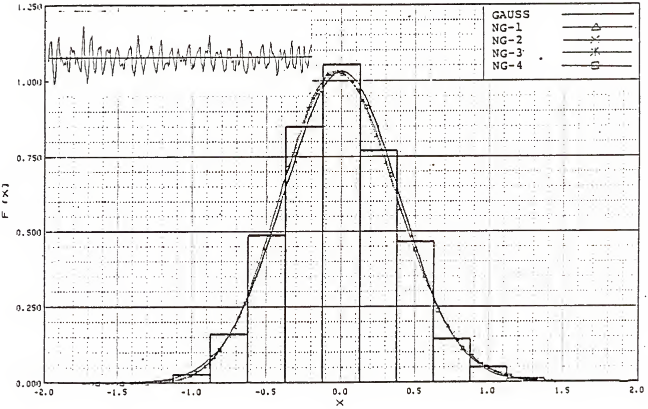


Figure 3.11 Histogram, probability distribution and portion of wave record for significant wave height 1.62 m at average water depth 6.97 m

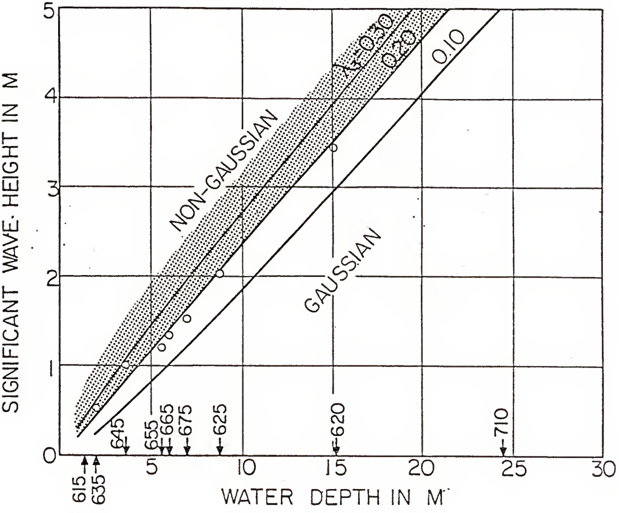


Figure 3.12 Largest significant wave height below which non-Gaussian characteristics are expressed as a function of water depth

0.2 can be considered as a Gaussian random process regardless of water depth.

3.4 Statistical Characteristics of Wave Period

Evaluation of the statistical characteristic of wave periods in finite water depth from the information of wave spectrum is extremely important for design consideration of coastal structures.

For waves in deep waters which are considered to be a Gaussian random waves, the average zero-crossing wave period is evaluated most commonly by the following equation developed by Rice (1945). That is

$$\bar{T}_{oc} = 2\pi \sqrt{\frac{m_0}{m_2}} \qquad (3.4)$$

where

$\bar{T}_{oc}$: computed average zero-crossing wave period

$m_n = \int_0^\infty w^n S(w)dw$: nth moment of wave spectrum

$S(w)$: wave spectrum

Since the waves in finite water depth show non-Gaussian properties, the currently available method for evaluating the period may no longer be valid. Collins (1967), Chakrabarti and Snider (1974), and Chakrabarti and Cooley (1977) found from the analysis of storm wave records that the average zero-crossing wave periods computed by Equation (3.4) were less than those measured from the wave record. Goda (1974) analyzed the wave data in finite water depth and evaluated that the average zero-crossing wave period computed from the spectrum by Equation (3.4) was 20% less than that measured from wave records.

To clarify the statistical characteristics of wave period in finite water depths, an analysis is carried out on measured wave records obtained during a storm at water depths ranging from 1.35 m to 8.77 m. That is, the average zero-crossing wave periods are evaluated by Equation (3.4) and the results are compared with the average periods obtained directly from wave records. The latter is denoted by $\overline{T}_{om}$

The comparisons of $\overline{T}_{oc}$ and $\overline{T}_{om}$ made at water depths of 1.35 m, 3.70 m, and 8.77 m in various sea severities are shown in Figure 3.13. Included also in this figure are the average lines drawn through the data.

It can be seen in the figure that,

(1) In general, the zero-crossing wave periods computed by Equation (3.4), $\overline{T}_{oc}$, are smaller than that measured from the wave record, $\overline{T}_{om}$. $\overline{T}_{oc}$ appears to agree with $\overline{T}_{om}$ only for small periods (2 sec or less) irrespective of water depth.

(2) The discrepancy between the magnitude of $\overline{T}_{oc}$ and $\overline{T}_{om}$ is particularly pronounced for waves in shallow

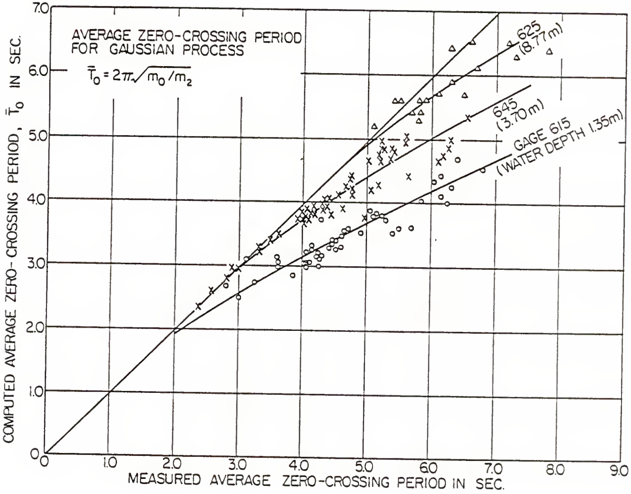


Figure 3.13 Comparison between the average zero-crossing wave period $\overline{T}_{oc}$ and $\overline{T}_{om}$

water. The discrepancy is reduced with increase in water depth, and $\overline{T}_{oc}$ appears to agree with $\overline{T}_{om}$ in deep water.

(3) Since the wave period generally increases with the increase of sea severity (significant wave height), it can be said that the difference in magnitude between $\overline{T}_{oc}$ and $\overline{T}_{om}$ increases with the increase of sea severity at a given water depth.

Based on the average lines shown in Figure 3.13, the ratios of $\overline{T}_{oc}$ and $\overline{T}_{om}$ are computed and plotted against the water depth as a function of the computed value $\overline{T}_{oc}$ as shown in Figure 3.14. This figure may serve to provide information as to the limiting water depth above which the computed average periods agree with the measured ones. For example, the average period of 5 sec. computed by Equation (3.4) can be used for design if the water depth of the location is greater than 9 meters. Otherwise the computed value has to be multiplied by the factor read off from the curve given in the figure.

In summary, the average zero-crossing wave period evaluated from the information of wave spectrum by Equation (3.4) is smaller than that measured from the wave record, in general. The difference in magnitude between them depends on the water depth as well as sea severity.

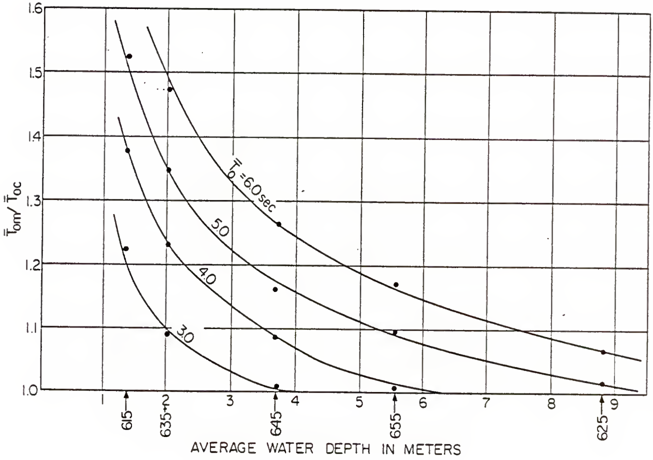


Figure 3.14 The ration $\overline{T}_{om}/\overline{T}_{oc}$ as a function of water depth

CHAPTER IV

STATISTICAL PREDICTION OF COASTAL WAVE HEIGHTS

In general, waves in deep water are commonly considered to be a Gaussian random process with a narrow-banded spectrum; and hence, the statistical characteristics of wave amplitudes and wave heights can therefore be predicted through the Rayleigh distribution.

However, one of the most distinct statistical characteristics of waves in finite water depth is that time histories show a definite excess of high crests and shallow troughs and cannot be considered a Gaussain random process. Hence, the current method for predicting wave heights based on the concept of Gaussian random process is no longer valid for probabilistic prediction of wave heights in finite water depth.

Furthermore, the magnitude of wave peaks in finite water depth is generally much greater than that of troughs. Therefore, probabilistic characteristics of peaks and troughs for waves in finite water depth should be evaluated independently in contrast to the prediction method for ocean waves in which probabilistic characteristics of peaks and troughs are assumed to be the same.

This chapter presents a method to statistically predict peaks, troughs, and peak-to-trough excursions (wave heights) of waves in finite water depth. The probability density functions are developed based on a non-Gaussian probability distribution with parameters which depend on the sea severity (significant wave height) and water depth.

This chapter consists of three sections. The first two sections discuss the method for predicting peaks and troughs as well as peak-to-trough excursions of waves. The third section presents the application of this prediction method for estimating the significant value and extreme value of peaks, troughs and peak-to-trough excursions of waves in a given sea severity (significant wave height) at a specified water depth. Numerical computations are carried out for various water depths ranging from 1.35 m to 8.77m. The results are compared with measured data obtained during the ARSLOE Project.

4.1 Probability Distribution of Peaks and Troughs

As stated, wave profiles observed in finite water depth show a definite excess of high crests and shallow troughs. Hence, the probability density function applicable to peaks and troughs should be derived independently. We first consider the probability density function for the peaks.

In this study, we assume that the wave spectrum is narrow-banded and hence there exists only a single peak during every half-cycle period. The probability that the peak, which is a random variable denoted by Ξ_+ exceeds a specified level ξ_+ is given as the expected value of the ratio of number of peaks exceeding the level ξ_+. to the number of peaks in the positive domain. That is

$$\Pr\left\{ \Xi_+ > \xi_+ \right\} = 1 - F(\xi_+) = E\left(\frac{N_{\xi+}}{N_+} \right) = \frac{\overline{N}_{\xi+}}{\overline{N}_+} \qquad (4.1)$$

where $F(\xi_+)$ = cumulative distribution function of Ξ_+

N_{ξ_+} = number of peaks exceeding a level ξ_+ per unit time.

N_+ = number of peaks on the positive domain per unit time.

$\overline{N}_{\xi_+}$ = expected value of N_{ξ_+}

$\overline{N}_+$ = expected value of N_+

It is noted that the expected value $E[\, N_{\xi+}/N_+ \,]$ is equal to the ratio of the individual expected values, denoted by $\overline{N}_{\xi+}$ and $\overline{N}_+$ respectively, only if we assume that the random variables ($N_{\xi+}/ N_+$) and N_+ are statistically independent. However, in general, this assumption appears to be acceptable for wind-generated waves.

For a narrow-banded random process, the expected values of $\bar{N}_{\xi+}$ and $\bar{N}_{+}$ can be approximately evaluated from the joint probability density function of wave surface elevation and velocity (time derivative of wave surface elevation). They are

$$\bar{N}_{\xi+} = \int_{-\infty}^{\infty} |\dot{x}|\, f(\, \xi_{+},\ \dot{x}\ )\, d\dot{x}$$

$$\bar{N}_{+} = \int_{-\infty}^{\infty} |\dot{x}|\, f(\, 0,\ \dot{x})\, d\dot{x} \tag{4.2}$$

From (4.1) and (4.2), the probability density function of ξ_{+} can be derived as

$$f(\xi_{+}) = \frac{-\dfrac{d}{d\xi_{+}} \displaystyle\int_{-\infty}^{\infty} |\dot{x}|\, f(\xi_{+},\ \dot{x}\ )\, d\dot{x}}{\displaystyle\int_{-\infty}^{\infty} |\dot{x}|\, f(\ 0,\ \dot{x}\ )\, d\dot{x}} \tag{4.3}$$

In the joint probability density function of wave surface elevation and velocity shown in Equation(4.3), the following assumptions are considered:

(1) The wave surface elevation and the velocity are statistically independent. To examine this assumption, a

statistical analysis on the measured wave profiles and velocities obtained by differentiating the wave profile has been carried out. In this analysis, the correlation coefficient of the surface elevations and velocities is evaluated.

Table 4.1 shows an example of correlation coefficients evaluated at three locations of different water depths. The table shows that the magnitude of the correlation coefficients of x and $\dot{x}$ are very small (on the order of 0.2 or less) regardless of water depth and sea severity. Hence, it may safely be assumed that wave surface elevation x and velocity $\dot{x}$ are statistically independent. This results in the joint probability density function of x and $\dot{x}$ shown in Equation(4.3) being expressed as the product of the individual probability density functions of $f(x)$ and $f(\dot{x})$. Here, the probability density function of wave surface elevation $f(x)$ is given in Equation (3.1). Based on the results of analysis on the effect of parameters on the probability density function presented in Chapter III, the terms with the parameters λ_3, λ_3^2, and λ_4 in the series are considered.

(2) The probability density function of velocity $f(x)$ is approximately a Gaussian distribution with zero mean and variance $\sigma_{\dot{x}}^2$. Results of a statistical analysis of velocity $\dot{x}$ obtained from differentiating measured wave profiles have indicated that the velocity approximately

Table 4.1 Correlation coefficient of wave surface elevation x and velocity $\dot{x}$

GAGE	WATER DEPTH (IN M.)	SIGNIFICANT WAVE HEIGHT (IN M.)	SKEWNESS	$\rho_{x\dot{x}}$
615	1.35	1.58	0.94	-0.191
615	1.35	1.83	1.00	-0.191
615	1.35	2.05	1.15	-0.184
645	3.70	2.10	0.50	-0.159
645	3.70	2.55	0.93	-0.155
625	8.77	2.93	0.38	-0.155
625	8.77	3.75	0.69	-0.137
625	8.77	3.63	0.71	-0.123

$\rho_{x\dot{x}}$: Correlation coefficient of elevation x and velocity $\dot{x}$

follows the Gaussian distribution. Examples of comparisons between the histograms and the Gaussian distribution of velocity are shown in Figure 4.1. The figure pertains to waves at a water depth of 3.70 m (12.1 ft.) in a sea of significant wave height 2.45 m (8.04 ft.).

For the probability density function specifically applicable to peaks of non-Gaussian random waves, it is necessary to consider a variance which is affiliated only with the positive part (upper portion) of wave profile.This variance, denoted by σ_{x+}^2 can be evaluated by taking the second moment of the probability density function of the wave profile, f(x), truncated at x=0. That is,

$$\sigma_{x+}^2 = \frac{\int_0^\infty x^2 \, f(x) \, dx}{\int_0^\infty f(x) \, dx} \tag{4.4}$$

By applying the probability density function given in Equation (3.1) to Equation (4.4), the variance σ_{x+}^2 is expressed as a function of the parameter λ_3 and the variance σ_x^2 That is

$$\sigma_{x+}^2 = \left(\frac{6\sqrt{\pi} + 2\sqrt{2}\,\lambda_3}{6\sqrt{\pi} - \sqrt{2}\,\lambda_3} \right) \sigma_x^2 \tag{4.5}$$

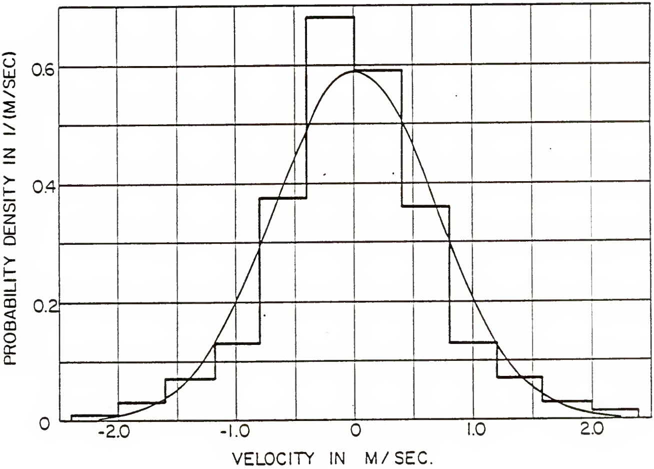


Figure 4.1 Comparison of wave velocity histogram and Gaussian distribution:
Average water depth 3.70 m, significant wave height 2.45 m

Taking these conditions into consideration, the probability density function of peaks can be obtained as follows:

$$\begin{aligned} f(\xi_+) = \frac{1}{L}\Bigg\{ & \frac{\xi_+}{\sigma_{x+}^2} e^{-\frac{1}{2}\left(\frac{\xi_+}{\sigma_{x+}}\right)^2} \left[1 + \frac{\lambda_3}{3!} H_3\left(\frac{\xi_+}{\sigma_{x+}}\right) + \frac{\lambda_4}{4!} H_4\left(\frac{\xi_+}{\sigma_{x+}}\right) + \frac{\lambda_3^2}{72} H_6\left(\frac{\xi_+}{\sigma_{x+}}\right)\right] \\ & - e^{-\frac{1}{2}\left(\frac{\xi_+}{\sigma_{x+}}\right)^2} \left(\frac{1}{\sigma_{x_+}}\right) \left[\frac{\lambda_3}{2!} H_2^*\left(\frac{\xi_+}{\sigma_{x+}}\right) + \frac{\lambda_4}{3!} H_3\left(\frac{\xi_+}{\sigma_{x+}}\right) + \frac{\lambda_3^2}{12} H_5\left(\frac{\xi_+}{\sigma_{x+}}\right)\right]\Bigg\} \end{aligned}$$

$$0 \leqslant \xi < \infty$$

where (4.6)

$$L = 1 + \frac{\lambda_4}{3!} - \frac{5}{24}\lambda_3^2$$

It is noted that the probability density function f(x) given in Equation (3.1) is not symmetric with respect to x = 0, and hence the differentiation involved in the numerator of Equation(4.3) results in a finite value of f(x) at $\xi_+ = 0$. To overcome this difficulty, the Hermite polyminal, $H_2^*\left(\frac{\xi_\pm}{\sigma_{x+}}\right)$, in Equation (4.6), is modified such that it becomes zero for small ξ_+. That is

$$H_2^*\left(\frac{\xi_+}{\sigma_{x+}}\right) = \begin{cases} \left(\frac{\xi_+}{\sigma_{x+}}\right) - 1 & \text{for } \frac{\xi_+}{\sigma_{x+}} \geqslant 1 \\ 0 & \text{for } \frac{\xi_+}{\sigma_{x+}} < 1 \end{cases} \tag{4.7}$$

The probability density function of troughs can be derived through the same procedure used for the derivation of the probability density function of wave peaks. The form of the probability density function of troughs is like that given in Equation (4.6) except ξ_+ should be negative and the variance σ_{x+}^2 should be replaced with the variance σ_{x-}^2 which is affiliated with the negative part (lower portion) of wave profile. The variance σ_{x-}^2 is given by

$$\sigma_{x-}^2 = \frac{\int_{-\infty}^{0} x^2\ f(x)\ dx}{\int_{-\infty}^{0} f(x)\ dx} = \left(\frac{6\sqrt{\pi} - 2\sqrt{2}\ \lambda_3}{6\sqrt{\pi} + \sqrt{2}\ \lambda_3}\right)\sigma_x^2 \tag{4.8}$$

Then, the probability density function of troughs is given as follows:

$$f(\xi_-) = \frac{1}{L}\left\{ \frac{\xi_-}{\sigma_{x-}^2} e^{-\frac{1}{2}\left(\frac{\xi_-}{\sigma_{x-}}\right)^2} \left[1 + \frac{\lambda_3}{3!} H_3\left(\frac{\xi_-}{\sigma_{x-}}\right) + \frac{\lambda_4}{4!} H_4\left(\frac{\xi_-}{\sigma_{x-}}\right) + \frac{\lambda_3^2}{72} H_6\left(\frac{\xi_-}{\sigma_{x-}}\right)\right] \right.$$
$$\left. - e^{-\frac{1}{2}\left(\frac{\xi_-}{\sigma_{x-}}\right)^2} \left(\frac{1}{\sigma_{x-}}\right)\left[\frac{\lambda_3}{2!} H_2^*\left(\frac{\xi_-}{\sigma_{x-}}\right) + \frac{\lambda_4}{3!} H_3\left(\frac{\xi_-}{\sigma_{x-}}\right) + \frac{\lambda_3^2}{12} H_5\left(\frac{\xi_-}{\sigma_{x-}}\right)\right]\right\}$$

$$-\infty < \xi_- \leqslant 0 \tag{4.9}$$

where

$$L = -\left(1 + \frac{\lambda_4}{3!} - \frac{5}{24}\lambda_3^2\right)$$

The Hermite polyminal $H_2^*\left(\frac{\xi_-}{\sigma_{x-}}\right)$ shown above, has to be modified in order to let it be zero for small ξ_-. That is

$$H_2^*\left(\frac{\xi_-}{\sigma_{x-}}\right) = \begin{cases} \left(\frac{\xi_-}{\sigma_{x-}}\right) - 1 & \text{for } \frac{\xi_-}{\sigma_{x-}} \leqslant -1 \\ 0 & \text{for } \frac{\xi_-}{\sigma_{x-}} > -1 \end{cases} \tag{4.10}$$

Note that if the parameters λ_3 and λ_4 are zero, then the probability density functions of peaks and troughs, shown in Equations (4.6) and (4.9) respectively, become identical and reduce to the Rayleigh probability distribution which is applicable to peaks and troughs of a Gaussian random process.

4.2 Probability Distribution of Wave Heights

The probability distribution of the peak-to-trough excursion, denoted by H, can be obtained as a distribution for the sum of two random variables: ξ_+ (peak) and ξ_- (trough). That is

$$H = \xi_+ + |\xi_-| \tag{4.11}$$

where H : peak-to-trough excursion (wave height)
ξ_+ : wave peak
ξ_- : wave trough

In the derivation of the probability density function of wave heights, peaks ξ_+ and troughs ξ_- are considered statistically independent. This assumption appears to be acceptable particularly for waves in shallow water. The results of analysis of waves measured at water depths of 1.35 m and 1.97 m have shown that the correlation coefficients computed between peaks and troughs are extremely small, on the order of 0.15 or less.

Thus, the joint probability density function of wave peaks and troughs is given as the product of the probability density functions of peaks and troughs given in Equations (4.6) and (4.9), respectively. The probability density function of peak-to-trough excursions can then be obtained by the following convolution integral;

$$f(\, H\,) = \frac{1}{K}\int_{0}^{H} f(\xi_{+})\; f(\;\xi_{+} - H\;)\; d\xi_{+} \tag{4.12}$$

where, K is a constant associated with the normalization of the probability density function, $f(H)$, and is given by

$$K = \int_{0}^{\infty} f(H)\; dH \tag{4.13}$$

By applying Equation (4.12), the significant wave height and extreme wave height can now be evaluated. The significant wave height, denoted by H_s, is given by

$$H_s = 3\int_{\xi_{*}}^{\infty} H\; f(H)\; dH \tag{4.14}$$

where ξ_{*} is the value which satisfies the following relationship:

$$\int_{0}^{\xi_{*}} f(H)\; dH = \frac{2}{3} \tag{4.15}$$

The probable extreme wave height in N observed waves, denoted by H_e, which is the modal value of the n-th order probability density function is given as the value which satisfies the following relationship,

$$\int_0^{H_e} f(H)\,dH = 1 - \frac{1}{N} \qquad (4.16)$$

It is noted that the significant value as well as extreme value of peaks and troughs of non-Gaussian waves also can be evaluated simply by replacing the probability density functions in Equation (4.14) and (4.16) with that of peaks and troughs shown in Equation (4.6) and (4.9).

4.3 Examples of Application

Before presenting examples of application of the method for predicting peaks, troughs and peak-to-trough excursion (include extreme value) of waves in finite water depth, the computation procedure will be summarized as follows:

Suppose we want to predict extreme wave height in a sea of significant wave height, H_s (in meters), at a location where the water depth is known as h (in meters). First, an evaluation is made to identify if waves in the specified sea condition can be considered as a Gaussian random process at this location. This can be done either by computing the skewness λ_3 by Equation (3.2) or by referring to the limiting line for the Gaussian random process given in Figure 3.11 . If skewness is less than or equal to 0.2, then the waves in the sea can be considered to be a Gaussian random process, and hence the prediction can be made by applying the commonly known method which is developed for evaluating wave height in deep water.

If the computed value of skewness, λ_3 is greater than 0.2, then the waves have to be considered as a non-Gaussian random process, and the probability density functions derived in the previous sections can now be applied for the prediction of peaks, troughs and wave heights.

In addition to the two parameters λ_3 and λ_4 which can be evaluated by Equation (3.2) and (3.3) respectively, the variance σ_x^2 is also necessary for computing the probability density functions. Since the relationship between the specified significant wave height and the variance σ_x^2 remains unknown in finite water depth, the following method is used;

(1) Let the relationship between H and variance σ_x^2 in finite water depth be expressed as $\sigma_x^2 = (H_s/C)^2$, where C is a constant. As an initial value of the variance, compute $\sigma_x^2 = (H_s/4.0)^2$ which is applicable to Gaussian random process. Then, evaluate σ_{x+}^2 and σ_{x-}^2 by Equation (4.5) and (4.8), respectively.

(2) Compute the probability density functions of wave peaks and troughs, shown in Equation (4.6) and (4.9), with the computed parameters λ_3 and λ_4 and σ_{x+}^2 or σ_{x-}^2 respectively.

(3) Compute the probability density function of wave height shown in Equation (4.12),and then the significant wave height is computed by Equation (4.14).

(4) Compare the computed significant wave height with the specified value. If these two values do not agree, then compute the variance again by adjusting the constant C. Repeat the procedure until the computed significant value agrees with the specified value.

(5) Significant value and extreme value of the peaks and troughs are computed from the final probability density function.

As an example of application, the probability density functions of peaks, troughs, and peak-to-trough excursions wave heights are computed at a water depth of 8.77 m in sea severity of significant wave height 3.39 m. The results of computation are compared with the measured data obtained during ARSLOE project as shown in Figure 4.2 through Figure 4.4. Included also in these figures is the Rayleigh probability distribution which is commonly used for waves in deep water.

As can be seen in Figure 4.2 and Figure 4.3, the probability density functions of peaks and troughs computed by Equations (4.6) and (4.9) respectively, agree reasonably

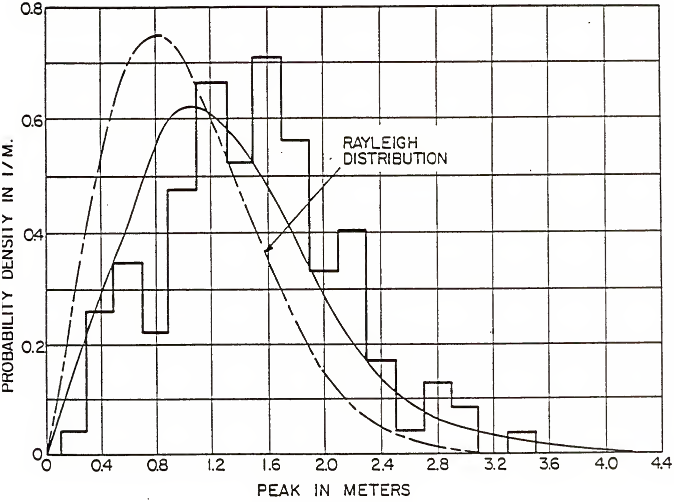


Figure 4.2 Comparison between observed histogram of peaks, Rayleigh distribution, and newly developed probability distribution: Average water depth 8.77 m, significant wave height 3.39 m

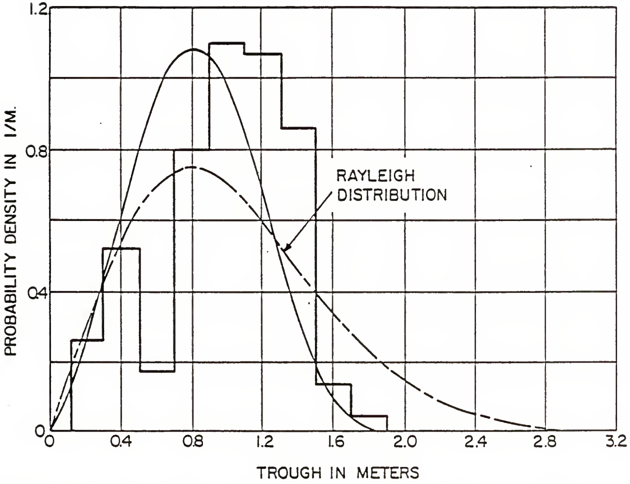


Figure 4.3 Comparison between observed histogram of troughs, Rayleigh distribution, and the newly developed probability distribution:
Average water depth 8.77 m, significant wave height 3.39 m

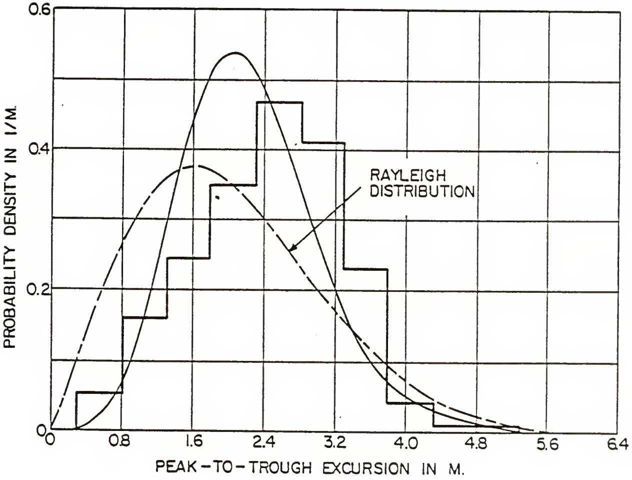


Figure 4.4 Comparison between observed histogram of peak-to-trough excursions, Rayleigh distribution, and the newly developed probability distribution: Average water depth 8.77 m, significant wave height 3.39 m

well with the histograms constructed from measured data, while the Rayleigh probability distribution substantially underestimates the peak values and overestimates the troughs value.

Figure 4.4 shows that the difference between histograms, Rayleigh probability distribution and the probability density function of peak-to-trough excursions is less pronounced than that observed for the peaks and troughs independently. But the probability density function computed by Equation (4.12) still shows a better agreement with the histogram than that of Rayleigh distribution.

Based on the probability density functions shown in Figures 4.2, 4.3 and 4.4, the significant value as well as extreme value can be predicted by applying Equation (4.14) and (4.16). Comparisons of predicted and measured significant value as well as extreme values for peaks, troughs, and peak-to-trough excursions (wave heights) is tabulated in Table 4.2. It is noted that the predictions are made for the same sea severity as the measured one; hence, the measured and predicted significant wave height are identical.

As can be seen in Table 4.2, the predicted significant and extreme values for peaks and troughs computed by the present method agree reasonably well with the measured values. However, there is a substantial difference between measured and predicted values computed by applying the Rayleigh probability distribution. More comparisons of the

Table 4.2 Comparison of observed and predicted significant and extreme value at average water depth 8.77 m

	PEAKS	TROUGHS	PEAK-TO-TROUGH EXCURSIONS
SIGNIFICANT VALUE			
Observed	2.21 m	1.36 m	3.39 m
Predicted by Present theory	2.32 m	1.36 m	3.39 m
Rayleigh distribution	1.70 m	1.70 m	3.39 m
EXTREME VALUE			
Observed	3.44 m	1.73 m	5.17 m
Predicted by Present theory	4.23 m	1.80 m	5.33 m
Rayleigh distribution	2.69 m	2.69 m	5.37 m

predicted and measured significant values as well as the extreme values are made at water depths of 1.35 m, 3.70 m and 8.77 m in various sea severities, and the results are tabulated in Table 4.3 and 4.4 respectively.

Table 4.3 shows that the significant values of peaks and troughs predicted by applying the probability distribution developed in the present study agree very well with those measured from wave records. On the other hand, the significant values of peaks predicted by applying the Rayleigh distribution are substantially lower than those measured from wave records, regardless of water depth and sea severity. Inversely, the predicted significant values of troughs computed by applying the Rayleigh distribution are much greater than those measured from wave records.

Table 4.4 shows that the extreme values of peaks and troughs predicted by the present method are also in better agreement with the measured extreme values than those predicted by applying the Rayleigh distribution. However, the magnitude of extreme peak-to-trough excursions predicted by applying the present theory is much greater than that of the observed excursions for waves in shallow water areas, say less than 4 meters. This may be attributed to wave breaking phenomenon frequently observed in shallow water areas. Since the present theory does not consider the breaking phenomenon, the computed extreme values of peak-to-trough excursions may be high for waves in shallow water area.

Table 4.3 Comparison of observed and predicted significant value

SIGNIFICANT VALUE (IN M.)

		PEAKS			TROUGHS			PEAK-TO-TROUGH EXCURSIONS
GAGE	WATER DEPTH (IN M.)	Observed	Present theory	Rayleigh dis.	Observed	Present theory	Rayleigh dis.	Observed
615	1.35	1.45	1.34	0.92	0.58	0.61	0.92	1.83
615	1.35	1.60	1.52	1.03	0.65	0.65	1.03	2.05
615	1.35	1.37	1.22	0.85	0.58	0.60	0.85	1.69
645	3.70	1.44	1.39	1.01	0.84	0.81	1.01	2.02
645	3.70	1.75	1.75	1.22	0.90	0.86	1.22	2.44
645	3.70	1.90	1.86	1.28	0.90	0.87	1.28	2.55
625	8.77	1.75	1.92	1.46	1.36	1.32	1.46	2.92
625	8.77	2.53	2.68	1.88	1.49	1.35	1.88	3.75
625	8.77	2.40	2.56	1.82	1.43	1.36	1.82	3.63

Table 4.4 Comparison of observed and predicted extreme value

EXTREME VALUE (IN M.)

		PEAKS			TROUGHS			PEAK-TO-TROUGH EXCURSIONS		
GAGE	WATER DEPTH (IN M.)	Observed	Present theory	Rayleigh dis.	Observed	Present theory	Rayleigh dis.	Observed	Present theory	Rayleigh dis.
615	1.35	2.09	2.58	1.47	1.01	0.83	1.47	2.68	3.13	2.95
615	1.35	2.39	2.98	1.64	0.88	0.88	1.64	2.63	3.55	3.28
615	1.35	1.96	2.33	1.36	0.88	0.78	1.36	2.08	2.83	2.72
645	3.70	2.41	2.50	1.65	1.19	1.05	1.65	2.77	3.15	3.30
645	3.70	2.70	3.33	1.97	1.36	1.15	1.97	3.15	4.05	3.94
645	3.70	2.90	3.50	2.02	1.22	1.15	2.02	3.53	4.25	4.04
625	8.77	2.69	3.20	2.37	1.68	1.73	2.37	3.90	4.18	4.74
625	8.77	3.79	4.98	2.97	2.25	1.78	2.97	5.12	6.10	5.94
625	8.77	4.32	4.65	2.85	2.27	1.78	2.85	5.70	5.75	5.69

Comparison results make it clear that the application of Rayleigh probability distribution for peaks of non-Gaussian random waves in finite water depth results in a substantial underestimation in the prediction of significant value as well as extreme value. This underestimation may be a serious problem for the design and safety operation of coastal structures, especially for those which are designed to preclude wave overtopping. In short, it is highly desirable that the probabilistic prediction of peaks and troughs of waves in finite water depth be based on the concept of the non-Gaussian random process.

CHAPTER V

CONCLUSIONS

This paper presents the results of a study on non-Gaussian characteristics of waves in finite water depths.The objectives of the study are (1) to clarify the non-Gaussian properties of coastal waves which depend on water depth as well as sea severity, and (2) to develop a method to predict peaks, troughs and peak-to-trough excursions of waves.

To achieve the first objective, a statistical analysis is carried out on wave time histories obtained at various water depths during a storm by the Coastal Engineering Research Center (CERC) in the Atlantic Ocean Remote Sensing Land-Ocean Experiment (ARSLOE) Project. From the results of statistical analysis, the following conclusions are drawn:

(1) Time histories of coastal waves show a definite excess of high crests and shallow troughs, in general, and waves are considered to be a non-Gaussian random process. The non-Gaussian properties depend on water depth and sea severity.

(2) The results of analysis of data show that the non-Gaussian characteristics of wave profiles can adequately be represented by the probability density function developed

by Longuet-Higgins which is expressed in a series form with parameters λ_3, λ_4, λ_5, etc. It is found in this study that the parameter which represents the skewness of the wave profile is the dominant parameter affecting the non-Gaussian characteristics of coastal waves, and that the combination of terms with the parameters λ_3, λ_3^2, and λ_4 (where λ_4 = kurtosis -3) in the probability density function best represents coastal wave profiles.

(3) In order to obtain the non-Gaussian properties as a function of water depth and sea severity, the parameter λ_3 is expressed as a function of water depth and sea severity. Furthermore, the functional relationship between the parameters λ_3 and λ_4 is derived.

(4) The coastal waves may still be considered to be a Gaussian random process if the sea severity is mild. It is found that the shape of the non-Gaussian probability density function with the value of λ_3 less than 0.2 is very close to that of a normal distribution. Hence, the coastal waves for which the parameter λ_3 (skewness) is less than 0.2 can be considered as a Gaussian random process regardless of water depth. Based on this, it is possible to evaluate the limiting sea severity below which the coastal waves are considered to be Gaussian for a given water depth.

(5) The average zero-crossing periods of coastal waves evaluated from the most commonly used formula (developed based on a Gaussian random process assumption) are smaller than those measured directly from wave records. This

difference in magnitude between them is particularly pronounced in shallow water and severe seas.

For the second objective of this study, the probability density functions applicable to peaks, troughs and peak-to-trough excursions of waves in finite water depth are developed based on the findings obtained in the first phase of the study. The results of numerical computations carried out at various water depths for various sea severities show that the probability density functions of peaks and troughs both reasonably agree with the histograms constructed from measured data. The Rayleigh probability density function which is commonly used for predicting amplitudes and heights of waves in deep water substantially underestimates the magnitude of peaks and overestimates the magnitude of troughs of coastal waves.

APPENDIX

WAVE DATA

The wave data used in the present study were obtained by the Coastal Engineering Research Center (CERC), U.S. Army, at its Field Research Facility, located at Duck, North Carolina. The Field Research Facility has a 550-meter-long research pier extending into the Atlantic Ocean, equipped with seven Baylor resistance-type wave gages (see Figure A.1). Extensive wave measurements were made at this site for two months in 1980 under the Atlantic Ocean Remote Sensing Land-Ocean Experiment (ARSLŒ) Project.

During October 23-25, an extratropical cyclon moved directly through the experimental area with wind speeds on the order of 10-15 m/sec. Continuous wave records were taken by the CERC Field Research Facility during the storm. The wave data analyzed in the present study are those recorded at the seven locations along the research pier and an additional two buoys (wave riders) located offshore along the extended line of the pier. The location of the wave gages and wave riders as well as the average water depth measured on October 21 and 27 by CERC are given in Figure A.1.

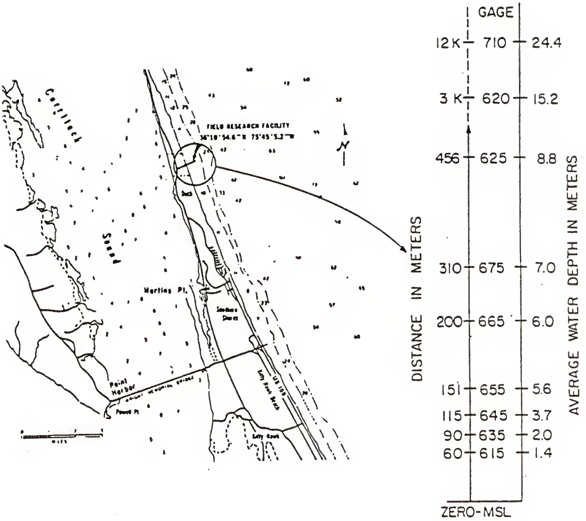


Figure A.1 Coastal Engineering Research Center (CERC) Field Research Facility, Duck, North Carolina (By courtesy of CERC)

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BIOGRAPHICAL SKETCH

Wei-Chi Wang was born July 19, 1955, in Kaung-Shung, Taiwan. He received his Bachelor of Science degree in hydraulic engineering from National Cheng-Kung University, Tainan, Taiwan, in 1977. He then enrolled as a graduate student in the National Cheng-Kung University. In 1979, he graduated with Master of Science degree in hydraulic and ocean engineering. In August, 1981, he entered graduate school at the University of Florida in the Department of Coastal and Oceanographic Engineering . He has worked as a research assistant while earning the Doctor of Philosophy degree.

On September 18, 1982 he married Marian Hong in Gainesville, Florida. They are expecting their first baby in September 1985.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a disseration for the degree of Doctor of Philosophy.

Michel K. Ochi

Michel K. Ochi, Chairman
Professor of Engineering Science, and Coastal and Oceanographic Engineering

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a disseration for the degree of Doctor of Philosophy.

Robert G. Dean

Robert G. Dean
Graduate Research Professor of Coastal and Oceanographic Engineering

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a disseration for the degree of Doctor of Philosophy.

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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a disseration for the degree of Doctor of Philosophy.

Edward K. Walsh

Edward K. Walsh
Professor of Engineering Science

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a disseration for the degree of Doctor of Philosophy.

J Hammack

Joseph L. Hammack
Professor of Engineering Science

This disseration was submitted to the Graduate Faculty of the College of Engineering and to the Graduate School, and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy

August 1985

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